## QUESTION 1

(a) A statistical mechanics treatment of molecules of a monatomic gas is usually made in terms of **microstates** and **macrostate** of the system. Explain the meanings of the highlighted terms in this statement. State the connection between them.

(b) Consider an ideal monatomic gas sample with molecules of mass, $m$, placed in a cubic box of side, $L$. A molecule of the gas would have velocity components, $v_x$, $v_y$, and $v_z$, respectively, in the Cartesian coordinate directions, $x$, $y$, and $z$. For a typical coordinate, $x$, say,

(i) Write down an expression for the momentum, $p_x$,

(ii) Use Heisenberg uncertainty principle to deduce an expression for the corresponding energy $E_x$ for this coordinate, in terms of an assigned quantum number, $n_x$, and the dimension of the box.

(iii) Hence write down expressions for the total energy of the molecule.

(iv) Given that the box is of side 10 cm, estimate a typical value for the number of quantum states, $n$, of the system, at temperature 300 K. {Hint: {You may use the postulate of **equal a priori probability distribution**, and the value of Boltzmann constant}.

## QUESTION 2

(a) Define the term **Partition Function**, used in the study of statistical mechanics. Using suitable examples, discuss the importance of the Partition Function in the study of statistical mechanics.

(b) The translational Partition Function, Z, for one molecule of an ideal monatomic gas in a container, is given by

$$Z = V\left[\frac{2\pi m k_B T}{h^2}\right]^{3/2}$$

(c) State what the symbols stand for in the above expression.

(d) Derive an expression for $\log_e Z$

(e) [illegible] 2 moles of helium gas in a container

[illegible]

## QUESTION 3

(a) In statistical mechanics we can treat a given system consisting of collections of **distinguishable**, or **indistinguishable** particles. Explain the meanings of the highlighted terms. Give one example of systems falling in each category.

(b) Explain the term **degeneracy**, used in the study of a system of indistinguishable particles in statistical mechanics.

(c) The total energy, $E$, of a hypothetical system of indistinguishable particles is given by $E^2 = n^2 h^2 f^2 = 66\, h^2 f^2$, where the symbols have their usual meanings, $n$ being the number of quantum states, which can have components $n_x$, $n_y$, and $n_z$, along the respective coordinate axes. Copy Table 1 below in your answer booklet. Using the lead provided in the first column, complete the table by finding values of $n_x$, $n_y$, and $n_z$, that satisfy the above energy relationship. Hence deduce the degeneracy of this hypothetical energy state.

**Table. 1**

$E^2 = n^2 h^2 f^2 = 66\, h^2 f^2$,

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n_x$ | 7 | | | | | | | | | | | | | |
| $n_y$ | 1 | | | | | | | | | | | | | |
| $n_z$ | 4 | | | | | | | | | | | | | |

deduce a numerical value of the entropy S for 2 moles in a Container or

**SECTION B:** *Answer any TWO questions*

1(a) Explain what you understand by the Ultraviolet Catastrophe.

(b) Four particles are to be distributed among four energy levels $\varepsilon_1 = 1, \varepsilon_2 = 2, \varepsilon_3 = 3, \varepsilon_4 = 4$ units having degeneracies $g_1 = 1, g_2 = 2, g_3 = 2, g_4 = 1$ respectively. The total energy of the system is 10 units. Find the possible distribution (macrostates) and the microstates corresponding to the most probable macrostates. Assume that the particles are: (i) distinguishable (ii) indistinguishable bosons (iii) indistinguishable fermions.

2(a) Explain the contributions of Wien, Raleigh and Jeans in the development of Blackbody radiation theory.

(b) A system has non-degenerate single-particle states with 0,1,2,3 energy units. Three particles are to bė distributed in these states such that the total energy of the system is 3 units. Find the number of microstates if the particles obey (i) MB statistics (ii) BE statistics (iii)FD statistics. Find the corresponding macrostates and microstates also.

3(a) Briefly explain the following:

(i) Maxwell Boltzmann Distribution

(ii) Bose- Einstein Statistics

(iii) Fermi-Dirac Statistics

(b) Three distinguishable particles (N = 3) (Yellow, Violet, Green) are to be *distributed in a 3-level* * system ($l$ =3) where first and second levels are non-degenerate1 but the second level is doubly degenerate $\{g_1, g_2, g_3\} = \{1, 2, 1\}$. Identify all possible microstates for the system when the occupancy sequence of the system is $\{n_1, n_2, n_3\} = \{1, 1, 1\}$.